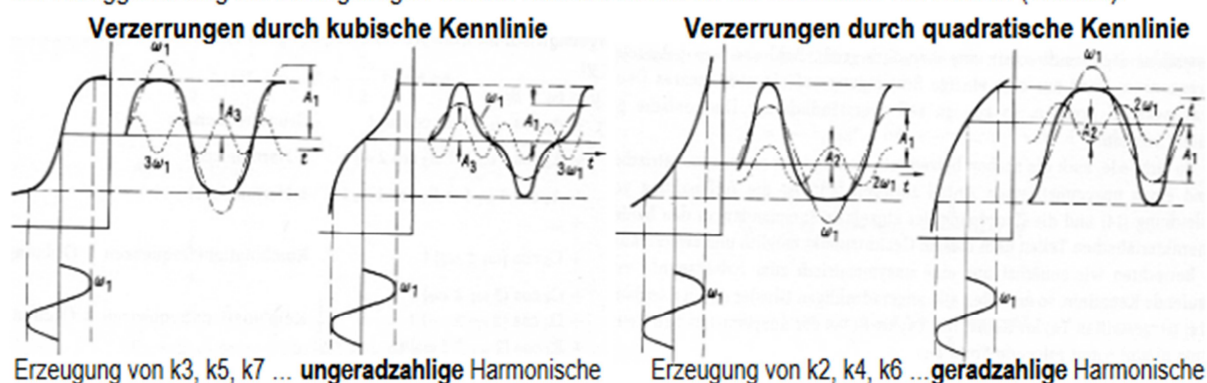# The Harmonic Distortion Myth

Some actual research investigations have motivated me to study the distortion topic more closely. With certain measurements the k2 distortion has been found to show up bigger amplitudes compared to k3 and higher harmonic distortions. So it this a good result as we can read in many publications? So I have decided to dive a bit deeper.

Sengpiel says about harmonic distortions:

Merke: So erklärt sich die Klarheit der Klänge mit hervortretenden geradzahligen Teiltönen. Das sind **Verzerrungen**, die klanglich vom subjektiven Hörempfinden nicht unbedingt abgelehnt werden, sondern sogar zur Klanggestaltung mit herangezogen werden können. Denke an die Beliebtheit von Röhren (Trioden).

Verzerrungen durch kubische Kennlinie — Erzeugung von k3, k5, k7 ... **ungeradzahlige** Harmonische

Verzerrungen durch quadratische Kennlinie — Erzeugung von k2, k4, k6 ... **geradzahlige** Harmonische

(Translation: **please note**: this explains the clarity of sound with even partial tones standing out. These are distortions, wich are not necessarily rejected by the subjective auditory sensation. Even more they are used for the sound design. Think about the popularity of tubes (triodes).)

So indeed you can find many places with statements about the positive influences of the k2 distortion.

This has led me to the question if it would be possible to get some benefit by adding k2 to a given music track. Something like adding a second voice one octave higher. This should perfectly fit to the basic harmonics and thus possibly result in a wonderful enhanced sound.

Sengpiel also shows how such distortions can be generated by nonlinear curves. It is also possible to use a little maths to create such a proper curve. Starting by the cos formula of the double angle (a double frequency will cover the double angle in the same time) we get:
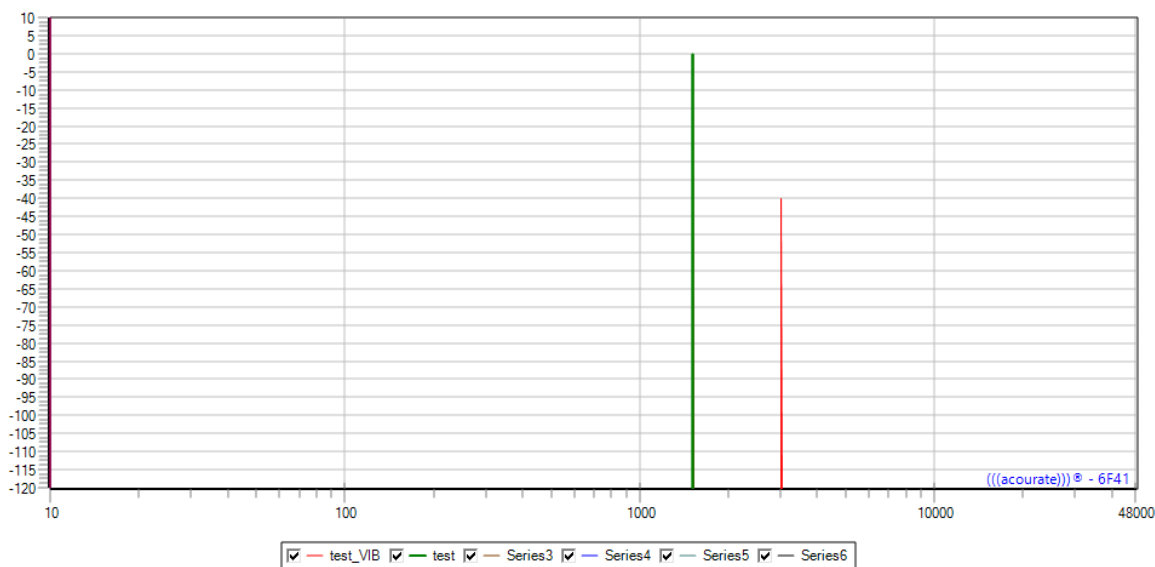
```
cos(2a) = 2*cos(a)² - 1
```

a polynom of 2nd degree

```
y = x + f*(2x² - 1)
```

The equation can be interpreted: based on the original signal we add the squared signal plus a constant, multiplied by the distortion ratio f.

It is no problem to apply DSP and so we can quickly check the result by a given sine wave as input signal. For a 1.5 kHz sine wave (samplerate 96 kHz) we get a frequency response:

The green spectral line at 1.5 kHz is the basic harmonic and we in addition the red spectral line at 3 kHz, here with -40 dB amplitude with a given distortion ratio f = 0.01

How does the result look like for a given mix of sine waves consisting of 375 Hz, 1.5 kHz und 12 kHz with same amplitude?
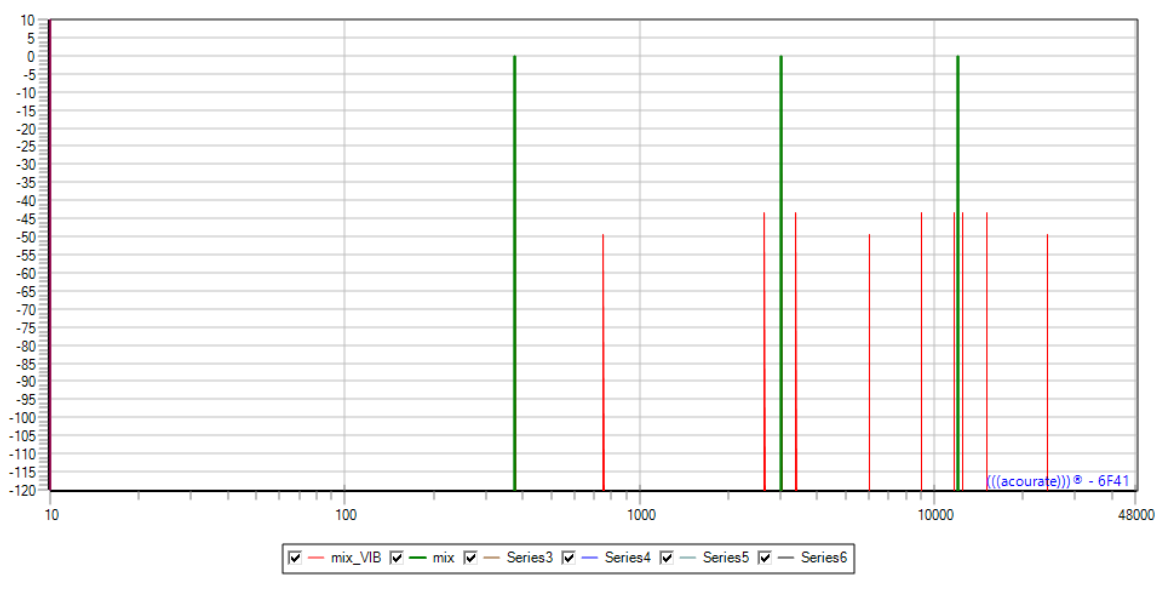
Beside the desired (or expected) harmonic frequencies 750 Hz, 3 kHz and 24 kHz there are sudden new frequencies which are totally non-harmonic. The reason can be found by studying the equation with an input signal consisting of a sum of two frequencies with x=a+b

```
y = a+b + f*(2(a+b)² - 1)
-> y = a+b + 2fa² +2fb² + 4fab -f
```

The equations shows up the term 4ab. It is equal to the multiplication of two frequencies (or to the amplitude modulation of a frequency by another frequency). Such a modulation creates the undesired side frequencies.The amplitude of these side frequencies is evewn by 6 dB higher than the intended double frequencies.

Uli Brüggemann, AudioVero   Aug. 2017

So we already suspect it: music isn't just a single frequency or a sum of two or three. Music is an arbitrary mixture of frequencies. Thus adding k2 does not result in pretty nice harmonic frequencies.

For demonstration I have prepared some short tracks:
1. original track http://www.acourate.com/freedownload/k2/TestOriginal.flac
2. the pure k2 part track with a constant distortion ratio f = -20 dB (for better audibility) http://www.acourate.com/freedownload/k2/Test_k2.flac
3. the k2 part track k2 with a frequency dependent boost (see following k2-picture of a recorded vinyl) http://www.acourate.com/freedownload/k2/Test_k2_boost.flac
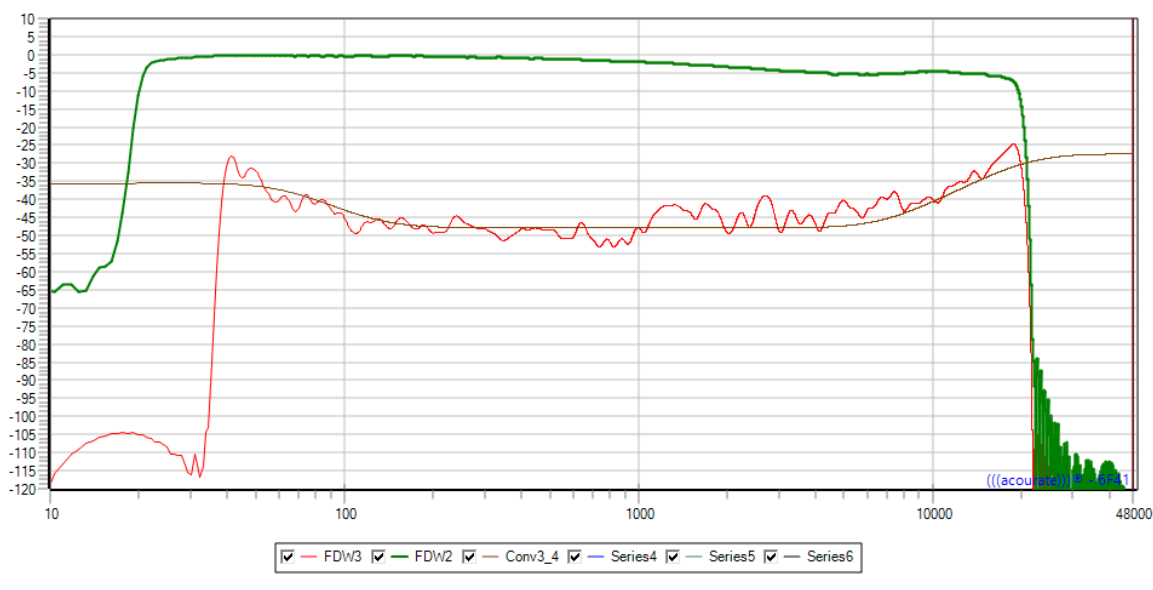
4. original track plus k2 nominal -40 dB: http://www.acourate.com/freedownload/k2/test_k2_boost40.flac
5. original track plus k2 nominal -20 dB: http://www.acourate.com/freedownload/k2/test_k2_boost20.flac

Summary: in ideal case we would expect to get the original music pitch shifted by an octave. But in real case we can hear that the **harmonic** distortion in a **myth**. Even with a simply k2 distortion we get non-linear harmonics.
Of course someone may argue that k2 is more nice to listen to than to k3 or higher. But this is not an argument against the non-harmonic results with any distortions.
So the best way is to avoid the distortions as much as possible.

Uli Brüggemann, AudioVero

PS: adding distortions by DSP must be done carefully
- a given track with a normalized amplitude range from -1 to +1 will result in a new range, e.g. -0.99 to +1.01 (k2 = -40 dB). Thus we get additional distortions by sample clipping
- with a given samplerate fs the allowed frequency range goes up to fs/2. It is necessary to apply a proper brickwall lowpass filter before calculating k2, k3 … to avoid aliasing = violation of Nyquist criteria